

# Empirical Period-Color and Amplitude-Color Relations for Classical Cepheids and RR Lyrae Variables

Anupam Bhardwaj<sup>1\*</sup>, Shashi M. Kanbur<sup>2</sup>, Harinder P. Singh<sup>1</sup>, Chow-Choong Ngeow<sup>3</sup>

*1. Department of Physics & Astrophysics, University of Delhi, Delhi 110007, India.*

*2. State University of New York, Oswego, NY, USA.*

*3. Graduate Institute of Astronomy, National Central University, Jhongli 32001, Taiwan*

Received on ; Accepted on

## ABSTRACT

We analyze Galactic, Large Magellanic Cloud and Small Magellanic Cloud Cepheids and RR Lyrae variables in terms of period-color (PC) and amplitude-color (AC) diagrams at the phases of maximum and minimum light. We compiled Galactic Cepheids  $V$ - and  $I$ -band data from the literature. We make use of optical bands light curve data from OGLE-III survey for Cepheids and RR Lyrae variables in the Magellanic Clouds. We apply the  $F$ -statistical test to check the significance of any variation in the slope of PC and AC relations for Cepheid variables. The PC relation at maximum light for Galactic Cepheids with periods longer than about 7 days is shallow and the corresponding AC relation is flat for the entire period range. For the fundamental mode Cepheids in the Magellanic Clouds, we find significant breaks in the PC and AC relations at both maximum and minimum light for periods around 10 days. The PC relation at maximum light for the Magellanic Clouds is flat for Cepheids with periods greater than 10 days. First overtone Cepheids with periods less than 2.5 days have a shallow PC relation at minimum light. For fundamental mode RR Lyraes, we confirm earlier work supporting a flat PC relation at minimum light and a significant relation between amplitude and color at maximum light. We find that no such relations exist for first overtone RR Lyrae stars. These findings are in agreement with stellar photosphere/hydrogen ionization front interaction considerations. These nonlinearities can provide strong constraints for models of stellar pulsation and evolution.

**Key words:** stars: variables: Cepheids - RR Lyrae - (galaxies:) Magellanic Clouds.

## 1 INTRODUCTION

Kanbur & Ngeow (2004) investigated the period-color (PC) and amplitude-color (AC) relations at the phase of maximum, mean and minimum light for fundamental mode Cepheids in our Galaxy, Large Magellanic Cloud (LMC) and Small Magellanic Cloud (SMC). These observational characteristics of Cepheids were analyzed following the work of Simon, Kanbur & Mihalas (1993), where full amplitude non-linear hydrodynamical pulsation models were used to explain the spectral class of Cepheids at maximum and minimum light. Simon, Kanbur & Mihalas (1993) applied the Stefan-Boltzmann law at maximum and minimum light to show that

$$\log T_{max} - \log T_{min} = \frac{1}{10}(V_{min} - V_{max}), \quad (1)$$

where  $T_{max}$  and  $T_{min}$  are defined to be the effective photospheric temperature at maximum and minimum light, respectively. If  $T_{max}$  or the color is independent of, or more weakly dependent on the pulsation period, then equation (1) predicts that there is a relation between the amplitude and the temperature at minimum light, and hence with color at minimum light. This relation suggests that higher amplitude stars have cooler temperatures and hence redder colors at minimum light.

Conversely, if  $T_{min}$  is independent or less dependent on the pulsation period, then the opposite is true: higher amplitude stars are driven to hotter temperatures and hence bluer colors at maximum light. Consequently a relation between the amplitude and the temperature at maximum light is expected.

\* E-mail: anupam.bhardwaj@gmail.com

In other words, if for some reason, the PC relation at maximum (or minimum) light is flat or shallow, then there is an AC relation at minimum (or maximum) light, suggesting a correlation between the amplitude and the extinction corrected color (Kanbur & Ngeow 2004; Kanbur, Ngeow & Buchler 2004). Further, if the PC relation at maximum and/or minimum light undergoes a significant change in slope at a given period, equation (1) suggests that the AC relation at minimum and/or maximum light should change somewhat at that period too. Galactic Cepheids have shallow PC relations at maximum light and an AC relation at minimum light (Simon, Kanbur & Mihalas 1993). In contrast, fundamental mode RR Lyrae stars have a flat PC relation at minimum light and a significant relation between amplitude and color at maximum light (Kanbur & Fernando 2005).

These studies on PC and AC relations were extended to observe multiphase characteristics by Ngeow & Kanbur (2006). These observational characteristics were further studied and published in a series of papers by Kanbur & Ngeow (2004), Kanbur, Ngeow & Buchler (2004), Kanbur & Ngeow (2006), Ngeow & Kanbur (2006), Kanbur, Ngeow & Feiden (2007) and Kanbur et al. (2010) for fundamental mode Cepheids.

One reason the color at a particular phase and period may be weakly dependent on stellar pulsation parameters, producing a flat or flatter PC relation at that phase could be due to the interaction of the stellar photosphere and hydrogen ionization front (HIF, Kanbur & Ngeow 2004). During a stellar pulsation cycle, the stellar photosphere and HIF move in and out of the mass distribution and are not necessarily co-moving. When the stellar photosphere and HIF are "engaged" (or the stellar photosphere lies at the base of the HIF), the temperature of the stellar photosphere is close to the temperature at which hydrogen ionizes and hence the color of the star is close to the color appropriate for the temperature at which hydrogen ionizes. The large rise in opacity due to the ionization of hydrogen makes it much harder for the stellar photosphere to be pushed further inside the mass distribution of the star. Put another way, the temperature needed to produce a given fraction of hydrogen ionization sufficient to prevent the photosphere from moving deeper inside the mass distribution is somewhat independent of global stellar parameters at low densities and temperatures. If this engagement occurs at higher densities, then the Saha ionization equation predicts that higher temperatures are needed to produce the same fractional hydrogen ionization and hence the same strength opacity wall. If the HIF and stellar photosphere are not engaged, then the temperature of the stellar photosphere will again be more dependent on global stellar parameters such as period.

Consequently, if for a group of stars, the stellar photosphere and HIF are engaged at low densities at a particular phase, this will then lead to a flat or flatter PC relation at that particular phase for this group of stars. For example, this is the case with Galactic fundamental mode Cepheids at maximum light or fundamental mode RR Lyraes at minimum light. As a fundamental mode Cepheid dims from maximum light, the HIF moves further back inside the star and the HIF and photosphere become disengaged. Thus the temperature of the photosphere or the color of the star becomes more dependent on global stellar parameters and thus on

the period. Therefore, the PC relation at minimum light for fundamental mode Galactic Cepheids is not flat. Very long period (periods greater than about 20 days) Cepheids have their HIF so far inside the mass distribution that the stellar photosphere and HIF are not engaged at any point during the pulsation cycle, even at maximum light.

In the case of fundamental mode RR Lyrae stars, as these stars brighten from minimum light, the HIF is driven further out in the mass distribution and so the stellar photosphere is pushed even further up the HIF, in order to achieve a given optical depth, say two-thirds. This necessitates a higher temperature that is more dependent on global stellar parameters and the stellar pulsation period.

The pulsation phase and period at which such HIF-stellar photosphere engagement/disengagement occurs varies with metallicity and input physics such as the mass-luminosity (ML) relation. This is because in hydrostatic models, the HIF moves further out in the mass distribution as the  $L/M$  ratio decreases and/or the  $T_{eff}$  increases. Thus more centrally concentrated stars such as RR Lyraes have their HIF further out in the mass distribution than Cepheids. Because the  $L/M$  ratio and  $T_{eff}$  determine the HIF's location, modeling of such observed nonlinearities appropriately has the potential to place strong constraints on stellar pulsation models using ML relations mandated by stellar evolution calculations.

The motivation for this paper was to extend the work on PC and AC relations with a much larger number of fundamental mode and first overtone Cepheids and RR Lyrae variables in the Magellanic Clouds based on the third phase of Optical Gravitational Lensing Experiment (OGLE-III) survey. We also significantly extend the number and nature of Galactic Cepheids included in our work over previous results. In particular we show that the empirical PCAC relations are reasonably consistent with the HIF and stellar photosphere interaction theory described above and in earlier papers. Further, we extend the analysis to include first overtone Cepheids and RR Lyraes found in the OGLE-III survey. The results are consistent with the stellar photosphere/HIF interaction theory outlined here and with the properties of the Saha ionization equation that is commonly used in models of stellar structure, pulsation and evolution.

In Section 2, we describe the Galactic Cepheid light curve data compiled from literature (Berdnikov 2008) and photometric light curve data for both Cepheid and RR Lyrae variables in the Magellanic Clouds taken from the OGLE-III survey (Soszyński et al. 2008; Soszyński et al. 2009; Soszyński et al. 2010a,b). In Section 3, we describe the method to evaluate the colors at the phases of minimum and maximum light from the Fourier fit to light curves. We also discuss the extinction correction in colors for all stars in Galaxy and Magellanic Clouds, and the  $F$ -statistical test for nonlinearity of the PC and AC relations. In Section 4, we discuss the variation of PC and AC relations for Cepheids and RR Lyraes separately in the LMC, SMC and the Galaxy. We compare our results for fundamental mode and first overtone stars and relate the changes with the theoretical models as described in the literature. We further discuss the robustness of our results in this section. We summarize the results and conclusions from this study in Section 5.

## 2 THE DATA

The light curve data for fundamental mode Galactic Cepheids in  $V$ - and  $I$ -band have been compiled from the catalogue of Berdnikov (2008). The photoelectric observations for  $V$ - and  $I$ -band in the Johnson and Cousins photometric systems were carried out by Berdnikov and his collaborators in series over nearly two decades between 1986 to 2004 (Berdnikov 1987, 1992; Berdnikov & Yakubov 1993; Berdnikov & Vozyakova 1995; Berdnikov, Ignatova & Pastukhova 1998; Berdnikov & Turner 2001, 2004a,b). We make use of 348 Galactic Cepheids, which are common in  $V$ - and  $I$ -band, in our analysis. This compilation is a significant improvement in terms of the number of Cepheids to that compiled in previous work (Kanbur & Ngeow 2004). The periods for the Galactic Cepheids were compiled from the McMaster catalogue of Galactic Classical Cepheids (Ferne et al. 1995).

The photometric light curve data in optical  $V$ - and  $I$ -band for Cepheids and RR Lyrae variables in the LMC and SMC was extracted from the OGLE-III survey. We make use of fundamental mode (FU) and first overtone (FO) Cepheids in LMC and SMC from this survey (Soszyński et al. 2008; Soszyński et al. 2010a), which include: 1802 FU and 1223 FO Cepheids in the LMC; and 2602 FU and 1629 FO Cepheids in the SMC as classified by OGLE-III survey that are used in our analysis. We also used the light curve data in optical bands for 26 long period fundamental mode Cepheids from OGLE-III Shallow Survey in the LMC (Ulaczyk et al. 2013). This increases our sample to 1828 fundamental mode Cepheids in the LMC.

Similarly, we also make use of fundamental mode RR Lyrae (RRab) and first overtone RR Lyrae (RRc) variables in the LMC and SMC from this survey (Soszyński et al. 2009; Soszyński et al. 2010b). There are 17334 RRab and 4871 RRc stars in the LMC, while there are 1917 RRab and 171 RRc stars in the SMC. These stars have photometric light curve data in both  $V$ - and  $I$ -band. We also used the time of initial epoch and the period as provided in OGLE-III database. We have summarized the data used in our analysis in Table 1.

Thus, one innovation in our paper is the large increase in data for Cepheids and RR Lyraes and the consideration of fundamental and first overtone mode in both types of pulsating variables.

## 3 THE METHOD

The Cepheid and RR Lyrae photometric light curve data in these three galaxies were then fitted with a Fourier sine series (Simon & Lee 1981; Deb & Singh 2010, 2014) with the following form:

$$m = m_0 + \sum_{k=1}^N a_k \sin(2\pi kx + \phi_k), \quad (2)$$

where,

$$x = \frac{(t - t_0)}{P} - \text{int} \left( \frac{(t - t_0)}{P} \right).$$

**Table 1.** The summary of the light curve data selected for the present analysis. These variables have light curve data in both  $V$ - and  $I$ -band.

	Variables	Mode	No. of stars	Reference
Galaxy	Cepheids	FU	348	B08
LMC	Cepheids	FU	1828	S08,U13
		FO	1223	S08
	RR Lyraes	FU	17334	S09
		FO	4871	-
SMC	Cepheids	FU	2602	S10a
		FO	1629	-
	RR Lyraes	FU	1917	S10b
		FO	171	-

**Notes:** The references in the last column are, B08 (Berdnikov 2008), S08 (Soszynski et al. 2008), S09 (Soszyński et al. 2009), S10a (Soszyński et al. 2010a), S10b (Soszyński et al. 2010b), U13 (Ulaczyk et al. 2013).

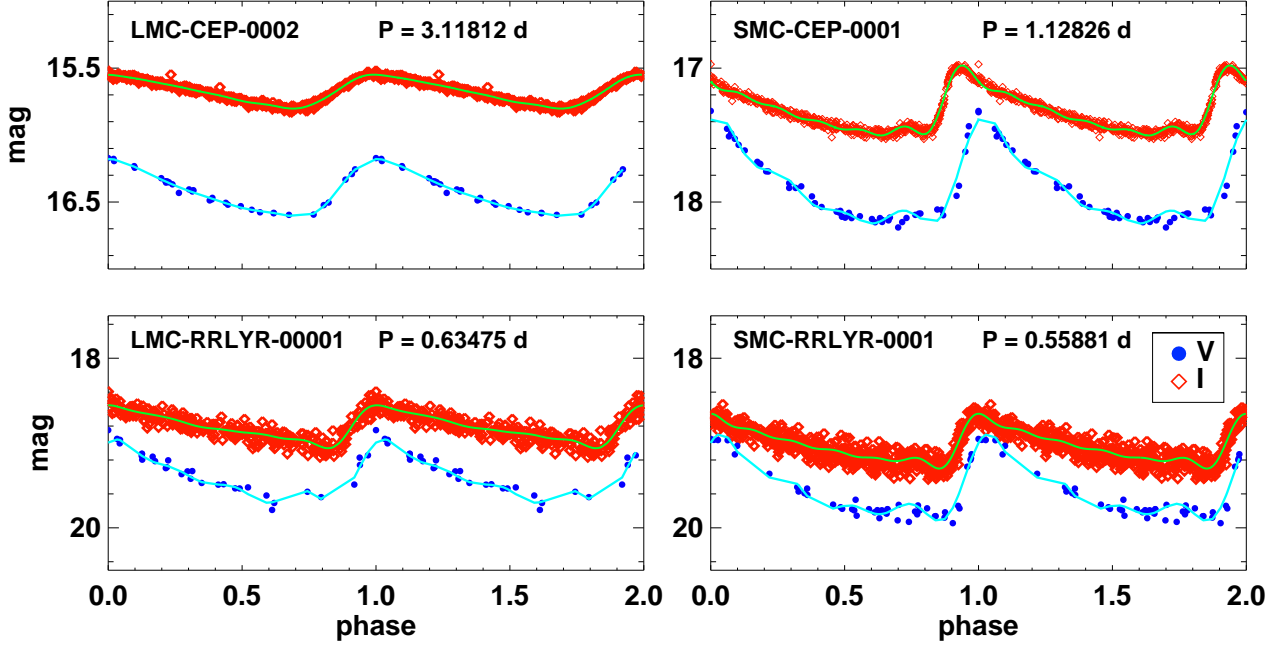
Here,  $t_0$  corresponds to the epoch of maximum brightness. This information is used to obtain a phased light curve that has maximum light at phase zero. Also,  $m_0$  is the mean magnitude,  $N$  is the optimum order of the fit and  $a_k$  and  $\phi_k$  are the Fourier amplitude and phase coefficients. A low value for  $N$  can lead to a systematic deviation from the best estimate of calculated parameters whilst a high value for  $N$  may lead to over-fitting and numerical ringing (Petersen 1986). So we varied the order of fit from 4 to 8 for each star and found the optimum order of fit corresponding to Bart's criterion (Bart 1982; Deb & Singh 2009). Examples of Fourier fitted light curves of a Cepheid and a RR Lyrae variable each in the LMC and the SMC are shown in Fig. 1. From the optimum order fit, we obtain the  $V$ -band amplitude and colors at maximum and minimum light defined as :

$$\begin{aligned} V_{amp} &= V_{min} - V_{max} \\ (V - I)_{max} &= V_{max} - I_{phmax} \\ (V - I)_{min} &= V_{min} - I_{phmin}. \end{aligned}$$

Here,  $I_{phmax}$  and  $I_{phmin}$  correspond to the  $I$ -band magnitude at the same phase as  $V_{max}$  and  $V_{min}$  (Kanbur & Ngeow 2004, 2006).

### 3.1 Extinction corrected color

The extinction correction for LMC and SMC stars was carried out using the extinction maps given in Haschke, Grebel & Duffau (2011, hereafter Haschke maps). We have used the RA/Dec position of the Cepheids and RR Lyraes from OGLE-III database to obtain the corresponding reddening from the Haschke maps. The color excess  $E(B - V)$  value for each Galactic Cepheid were taken from the Tammann, Sandage & Reindl (2003). Finally, the colors at these two extremum phases have been corrected for extinction using  $A_{V,I} = R_{V,I} E(B - V)$ . The values of  $R$  for Galactic data:  $R_V = 3.17$ ,  $R_I = 1.89$ , are obtained from Tammann, Sandage & Reindl (2003). Similarly for the LMC and SMC,  $R_V = 2.40$ ,  $R_I = 1.41$  were used with color excess  $E(V - I)$  values obtained from Haschke, Grebel & Duffau



**Figure 1.** Examples of Fourier fitted light curves of a Cepheid and a RR Lyrae variable each in the LMC and SMC from OGLE-III survey.

(2011). We apply the same extinction values of  $A_{V,I}$  to the colors at maximum and minimum light, since  $A_{V,I}$  is independent of pulsation phase.

### 3.2 The $F$ -statistical test

Once we have the reddening corrected colors, we plot them against  $\log(P)$  and  $V$ -band amplitudes to find PC and AC relations. To test for any possible breaks in the PC and AC relations, we use the  $F$ -test to determine the statistical significance of any variation in slope. We use the method described by Kanbur & Ngeow (2004) to fit a single regression line and a double regression line separately to the PCAC relation under consideration. Therefore, under the null hypothesis, we can fit a regression line over the entire period range. As an alternative hypothesis, we can fit two regression lines, one separately for stars having periods smaller/greater than the assumed break point. We will call the former case (single regression line) to be the reduced model as described in Kanbur & Ngeow (2004), while the latter (two regression lines) is the full model. We write the first model as:

$$Y = a + bX; \text{ Reduced model.} \quad (3)$$

This regression line is over the entire period range. To test a break at some point, we make use of the full model:

$$\begin{aligned} Y &= a_S + b_S X; \text{ where } P < \text{break point} \\ &= a_L + b_L X; \text{ where } P \geq \text{break point.} \end{aligned} \quad (4)$$

Here,  $Y$  is the dependent variable, i.e. the color ( $V-I$ ) while  $X$  is the independent variable, which is  $\log(P)$  in case of PC relations and  $V$ -band amplitude in case of AC relations.

Our interest is to see whether the slope for short/long

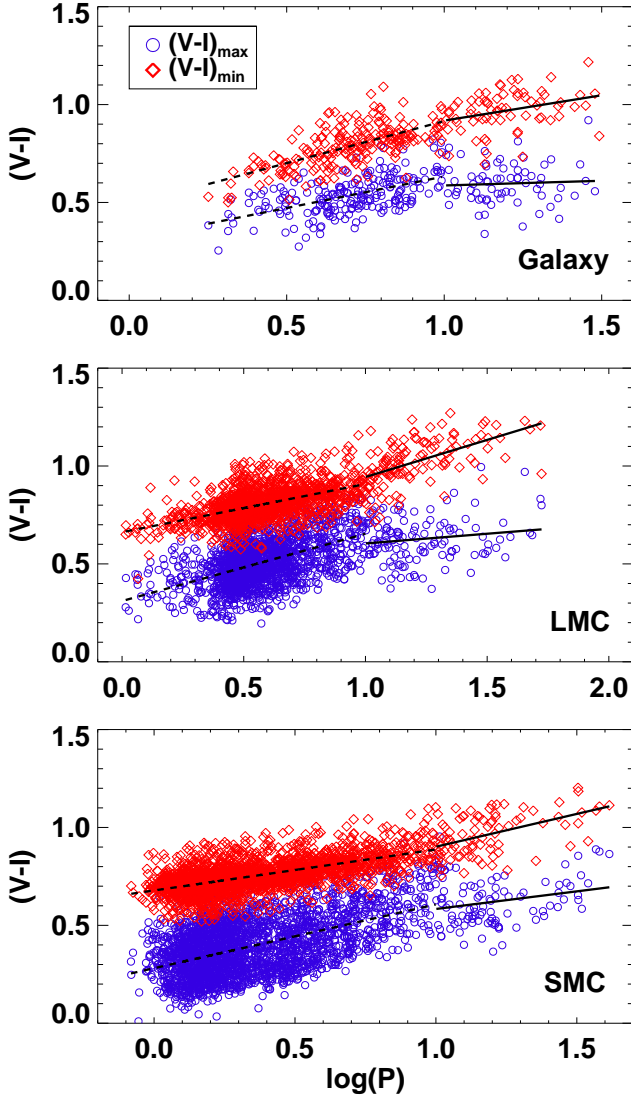
periods ( $b_S/b_L$ ) varies significantly for the break point under consideration. We can apply an appropriate  $F$  test statistic formulated as described in the following equation,

$$F = \frac{(RSS_R - RSS_F)/[(n-2) - (n-4)]}{RSS_F/(n-4)}, \quad (5)$$

where  $RSS_R$ ,  $RSS_F$  are the residual sums of squares in the single line regression (reduced model) and two line regression (full model), respectively. Here,  $n$  is the number of stars in the entire sample. The full model with two regression lines will have a smaller residual sum of squares. The null hypothesis that the single regression line is sufficient can be rejected if the calculated value of  $F$  is greater than the critical value. The critical value is referred to the value of the  $F$ -distribution at 95% confidence level, i.e.  $F_C = F_{2,n-4}$ . Thus, a large value of  $F$  provides evidence to support the rejection of the null hypothesis and hence the two regression line is a better fit to the data under consideration. The probability of the observed value of the  $F$  statistic,  $P(F)$ , under the null hypothesis, gives the significance of this reduction in sum of squares. In all the plots for the PC and AC relations, we remove the  $3\sigma$  outliers to have a robust regression analysis.

In all subsequent tables,  $a, b$  refer to the intercept and slope respectively and the subscripts  $all, S, L$  refer to the entire period range, short period range and long period range, respectively. The period separating short from long is mentioned in the table title.





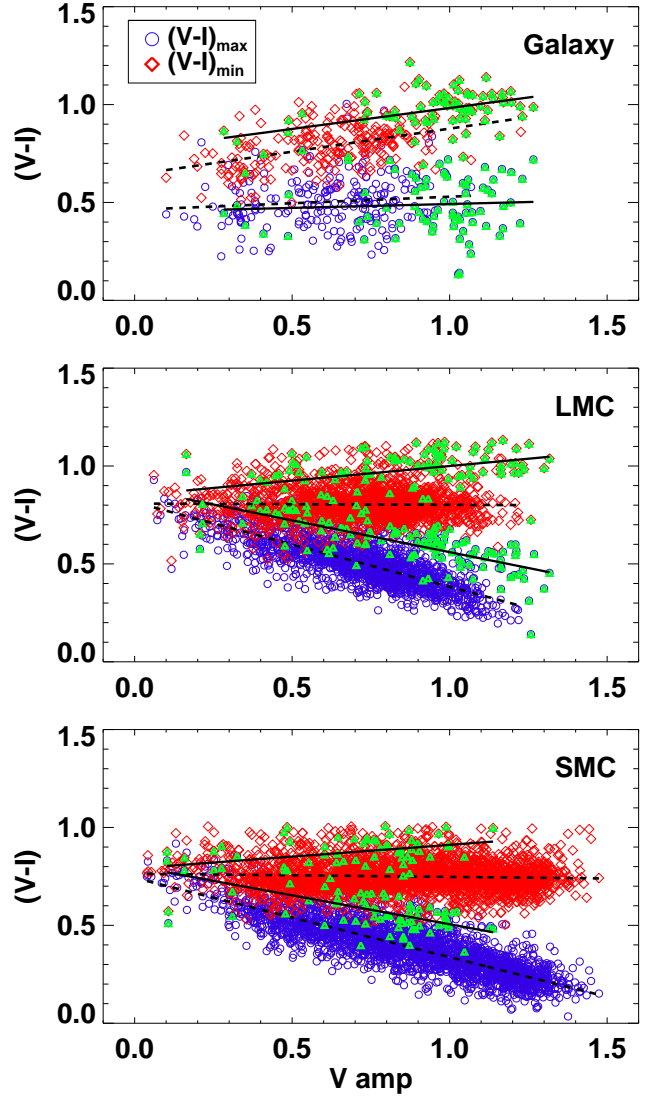
**Figure 2.** PC relations for fundamental mode Cepheids at maximum and minimum light for Galaxy, LMC and SMC. The dashed/solid lines represent the best fit to shorter/longer period Cepheids separated at 10 days.

## 4 ANALYSIS AND RESULTS

### 4.1 Period-Color & Amplitude-Color relations for Cepheid variables

We describe the PC and AC relations for FU and FO Cepheids in the Galaxy, LMC & SMC in this subsection. The plots of PC and AC relations for FU Cepheids are given in Fig. 2 and Fig. 3 respectively. Since, Kanbur & Ngeow (2004) observed a break in the LMC PC relation for periods greater than 10 days, we fit the PC and AC relations to the entire sample as well as the long and short period samples separated at 10 days. The slope and intercepts for these PC and AC relations in the Galaxy, LMC and SMC together with the results of  $F$ -test are provided in Table 2.

For Galactic FU Cepheids, the slope of the maximum light PC relation is significantly smaller for longer period



**Figure 3.** AC relations for fundamental mode Cepheids at maximum and minimum light for the Galaxy, LMC and SMC. Triangle represents the Cepheids having periods greater than 10 days in both the plots at maximum and minimum light. The dashed/solid lines represent the best fit to shorter/longer period Cepheids.

( $P > 10$  days) Cepheids than it is for their shorter period counterparts. In fact this is also true for a break period of 7 days (or  $\log(P) \sim 0.845$ ), as we see a flat PC relation at maximum light from a period of about 7 days instead of 10 days.  $F$ -test results for a break period at 7 days are presented in Table 3. Together with this we have a statistically significant change in the slope of the AC relation at minimum light at both periods: 7 or 10 days. The AC relation at maximum light does not change significantly at 7 or 10 days. These findings are consistent with equation (1).

For FU Cepheids in the Magellanic Clouds at maximum light, we observe a significant change in the slope of the PC relation between shorter ( $P < 10$  days) and longer ( $P > 10$  days) period Cepheids. We find a significantly flatter PC relation at maximum light for FU Cepheids with

**Table 2.** Results of F test on PC and AC relations for fundamental mode Cepheids to determine possible nonlinearities at 10 days.

Phase		$b_{(all)}$	$a_{(all)}$	$b_S$	$a_S$	$b_L$	$a_L$	F	P(F)
Galaxy									
PC	max	$0.177 \pm 0.021$	$0.406 \pm 0.018$	$0.322 \pm 0.034$	$0.311 \pm 0.024$	$0.048 \pm 0.107$	$0.539 \pm 0.128$	11.527	0.000
	min	$0.378 \pm 0.019$	$0.520 \pm 0.017$	$0.429 \pm 0.035$	$0.486 \pm 0.025$	$0.259 \pm 0.093$	$0.659 \pm 0.113$	2.142	0.119
AC	max	$0.024 \pm 0.033$	$0.483 \pm 0.025$	$0.067 \pm 0.049$	$0.463 \pm 0.032$	$0.040 \pm 0.075$	$0.452 \pm 0.072$	1.075	0.343
	min	$0.341 \pm 0.025$	$0.596 \pm 0.018$	$0.235 \pm 0.034$	$0.642 \pm 0.022$	$0.214 \pm 0.051$	$0.769 \pm 0.049$	24.128	0.000
LMC									
PC	max	$0.253 \pm 0.011$	$0.359 \pm 0.007$	$0.343 \pm 0.015$	$0.311 \pm 0.009$	$0.099 \pm 0.056$	$0.505 \pm 0.069$	32.022	0.000
	min	$0.300 \pm 0.007$	$0.633 \pm 0.005$	$0.246 \pm 0.010$	$0.662 \pm 0.006$	$0.379 \pm 0.051$	$0.565 \pm 0.063$	26.668	0.000
AC	max	$-0.389 \pm 0.010$	$0.794 \pm 0.007$	$-0.432 \pm 0.009$	$0.815 \pm 0.007$	$-0.324 \pm 0.028$	$0.884 \pm 0.026$	235.694	0.000
	min	$0.035 \pm 0.010$	$0.788 \pm 0.008$	$-0.006 \pm 0.010$	$0.808 \pm 0.007$	$0.149 \pm 0.034$	$0.851 \pm 0.030$	218.322	0.000
SMC									
PC	max	$0.302 \pm 0.008$	$0.288 \pm 0.004$	$0.326 \pm 0.011$	$0.281 \pm 0.004$	$0.180 \pm 0.073$	$0.404 \pm 0.088$	7.381	0.001
	min	$0.225 \pm 0.005$	$0.674 \pm 0.002$	$0.209 \pm 0.005$	$0.678 \pm 0.002$	$0.335 \pm 0.063$	$0.566 \pm 0.076$	13.870	0.000
Ac	max	$-0.408 \pm 0.005$	$0.749 \pm 0.005$	$-0.405 \pm 0.005$	$0.742 \pm 0.005$	$-0.296 \pm 0.041$	$0.801 \pm 0.033$	128.116	0.000
	min	$-0.019 \pm 0.006$	$0.769 \pm 0.005$	$-0.016 \pm 0.005$	$0.764 \pm 0.005$	$0.121 \pm 0.048$	$0.791 \pm 0.036$	72.935	0.000

**Table 3.** Results of F test on PC and AC relations for fundamental mode Cepheids to check period breaks at 7 days for Galactic Cepheids and at 2.5 days for SMC Cepheids.

Phase		$b_{(all)}$	$a_{(all)}$	$b_S$	$a_S$	$b_L$	$a_L$	F	P(F)
Galaxy									
PC	max	$0.177 \pm 0.021$	$0.406 \pm 0.018$	$0.286 \pm 0.052$	$0.333 \pm 0.033$	$0.042 \pm 0.046$	$0.552 \pm 0.049$	7.539	0.001
	min	$0.378 \pm 0.019$	$0.520 \pm 0.017$	$0.573 \pm 0.054$	$0.404 \pm 0.035$	$0.347 \pm 0.040$	$0.547 \pm 0.042$	7.065	0.001
AC	max	$0.024 \pm 0.033$	$0.483 \pm 0.025$	$0.041 \pm 0.061$	$0.476 \pm 0.039$	$0.023 \pm 0.045$	$0.480 \pm 0.037$	0.129	0.879
	min	$0.341 \pm 0.025$	$0.596 \pm 0.018$	$0.265 \pm 0.038$	$0.600 \pm 0.025$	$0.257 \pm 0.030$	$0.708 \pm 0.026$	37.111	0.000
SMC									
PC	max	$0.302 \pm 0.008$	$0.288 \pm 0.004$	$0.265 \pm 0.030$	$0.298 \pm 0.007$	$0.350 \pm 0.017$	$0.251 \pm 0.012$	6.162	0.002
	min	$0.225 \pm 0.005$	$0.674 \pm 0.002$	$0.176 \pm 0.016$	$0.685 \pm 0.003$	$0.263 \pm 0.009$	$0.645 \pm 0.007$	15.078	0.000
AC	max	$-0.408 \pm 0.005$	$0.749 \pm 0.005$	$-0.365 \pm 0.006$	$0.679 \pm 0.005$	$-0.419 \pm 0.007$	$0.816 \pm 0.006$	590.648	0.000
	min	$-0.019 \pm 0.006$	$0.769 \pm 0.005$	$0.015 \pm 0.006$	$0.709 \pm 0.006$	$-0.023 \pm 0.008$	$0.826 \pm 0.007$	431.693	0.000

periods greater than 10 days. In fact for LMC FU Cepheids, the longer period slope is consistent with zero to within the quoted errors. We note that there are only two stars having period greater than 100 days, which were not considered in the best fit PC relations. Further for SMC FU Cepheids, this shallow PC relation is observed for a small region of period range  $1.0 < \log(P) < 1.3$ . For stars with periods greater than 20 days, there was again a small but significant change in the slope of the PC relation. Now at minimum light, there is also a significant change in slope at 10 days for both the LMC and SMC. We also see that the short period Magellanic Clouds FU Cepheids PC relations at maximum and minimum light are almost parallel to each other.

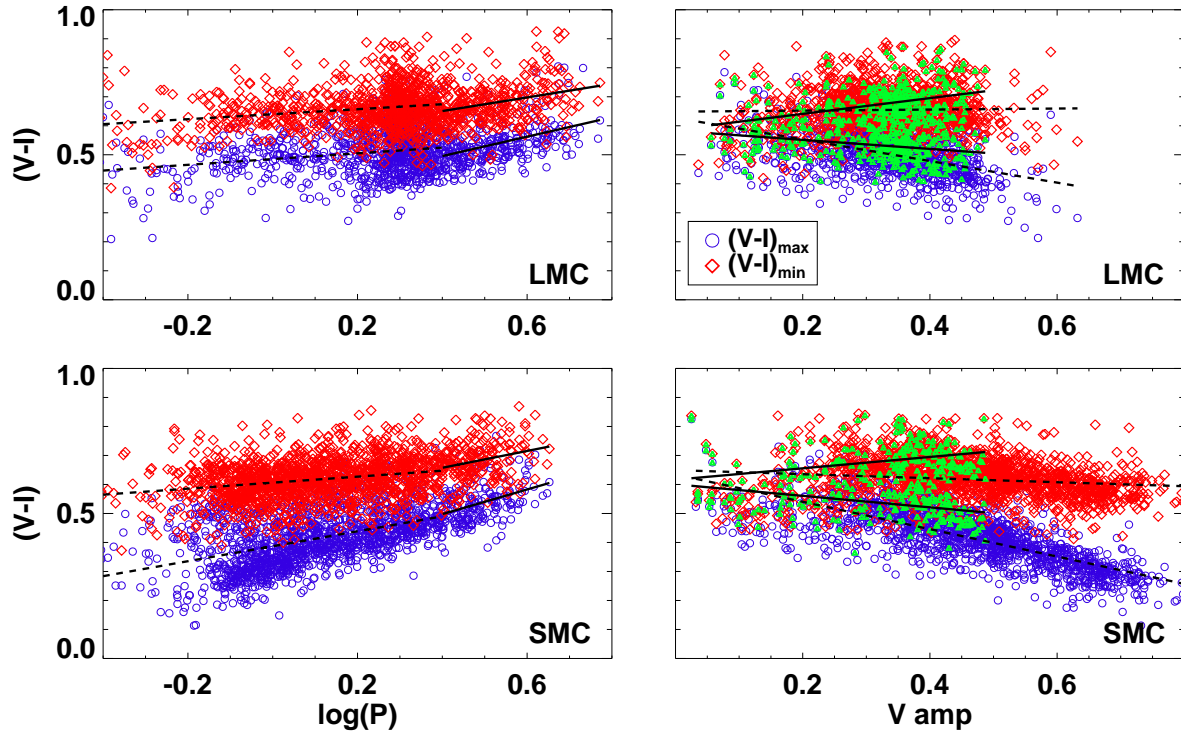
For Galactic FU Cepheids, the AC relation at minimum light has a non-zero slope indicating that higher amplitude stars are driven to cooler temperatures. Hence the colors become redder at minimum light because the PC relation at maximum light is flat in accordance with equation (1). In the case of LMC and SMC FU Cepheids, the overall AC relations exhibit a flat slope at minimum light but

a positive slope at maximum light such that higher amplitude stars are driven to hotter temperatures at maximum light. This is different to the Galactic AC relation at minimum light (see Fig. 3). These lower metallicity Cepheids in the Magellanic Clouds, increase their amplitude by getting hotter at maximum light. However, if we consider only the longer period ( $P > 10$  days) FU Cepheids in the Magellanic Clouds, we see a significant change in the AC slopes at minimum light at  $\log(P) \sim 1$  such that for  $\log(P) > 1$ , higher amplitude stars are driven to cooler temperatures and hence redder colors at minimum light, again in accordance with equation (1). However the slopes in the AC relations for long period ( $\log(P) > 1$ ) Cepheids are only marginally different from zero.

It has been known that the period-luminosity relation for SMC FU Cepheids exhibits a break at a period of 2.5 days (or  $\log(P) = 0.4$ , see Bauer et al. 1999). In addition, the plots of Fourier parameters against  $\log(P)$  for SMC Cepheids show a change in progression around this period (Simon & Lee 1981). We will comment on this further in the

**Table 4.** Results for F test on PC and AC relations for first overtone Cepheids to check period breaks at 2.5 days.

	Phase	$b_{(all)}$	$a_{(all)}$	$b_S$	$a_S$	$b_L$	$a_L$	F	P(F)
LMC									
PC	max	$0.105 \pm 0.008$	$0.484 \pm 0.003$	$0.099 \pm 0.011$	$0.485 \pm 0.003$	$0.334 \pm 0.042$	$0.362 \pm 0.022$	10.384	0.000
	min	$0.085 \pm 0.008$	$0.639 \pm 0.003$	$0.086 \pm 0.010$	$0.640 \pm 0.003$	$0.235 \pm 0.046$	$0.557 \pm 0.024$	5.216	0.006
AC	max	$-0.327 \pm 0.022$	$0.619 \pm 0.008$	$-0.373 \pm 0.026$	$0.627 \pm 0.009$	$-0.156 \pm 0.039$	$0.583 \pm 0.013$	24.107	0.000
	min	$0.071 \pm 0.023$	$0.636 \pm 0.008$	$0.018 \pm 0.026$	$0.649 \pm 0.009$	$0.271 \pm 0.040$	$0.587 \pm 0.014$	20.803	0.000
SMC									
PC	max	$0.281 \pm 0.010$	$0.387 \pm 0.002$	$0.258 \pm 0.013$	$0.387 \pm 0.002$	$0.426 \pm 0.079$	$0.327 \pm 0.039$	4.712	0.009
	min	$0.130 \pm 0.008$	$0.606 \pm 0.002$	$0.103 \pm 0.010$	$0.606 \pm 0.002$	$0.290 \pm 0.086$	$0.542 \pm 0.042$	9.593	0.000
AC	max	$-0.490 \pm 0.011$	$0.649 \pm 0.005$	$-0.472 \pm 0.012$	$0.635 \pm 0.006$	$-0.198 \pm 0.041$	$0.600 \pm 0.015$	85.904	0.000
	min	$-0.087 \pm 0.012$	$0.664 \pm 0.006$	$-0.069 \pm 0.012$	$0.649 \pm 0.006$	$0.196 \pm 0.042$	$0.616 \pm 0.015$	74.455	0.000

**Figure 4.** PC and AC relations for LMC and SMC first overtone Cepheids at maximum and minimum light. The triangles represent the stars having periods greater than 2.5 days in both the plots at maximum and minimum light. The dashed/solid lines represent the best fit regression lines to shorter/longer period Cepheids.

subsection describing robustness of our results. We present the results for PCAC breaks at a period of 2.5 days for SMC FU Cepheids in the second half of Table 3. These tests are highly significant at both maximum and minimum light. This analysis extends the earlier work of EROS collaboration (Bauer et al. 1999) to the PC relation of OGLE-III  $V$ - and  $I$ -band data.

Fig. 4 presents the PC and AC relations for LMC and SMC FO Cepheids. We also test the significance of any variation in slope of PCAC relations at 2.5 days for FO Cepheids in Magellanic Clouds. The slopes and intercepts of these PCAC relations, together with the  $F$ -test results, are sum-

marized in Table 4. For Magellanic Clouds FO Cepheids, PCAC relations provide evidence of a break at 2.5 days at both maximum and minimum light. We observed a flatter PC relation at minimum light and a greater slope at maximum light, for both LMC and SMC. The slope of the PC relation is almost flat for Cepheids having periods less than 2.5 days. The AC relation at maximum light has a significant negative slope while we observe a flat AC relation at the phase of minimum light for both LMC and SMC.

We also note that this behavior of a flatter slope at minimum light in the PC relation for FO Cepheids, particularly those with periods less than 2.5 days, is very similar to that

**Table 5.** Slope and intercepts for PC and AC relations for RRab and RRc stars at maximum and minimum light.

	Phase	slope(RRab)	intercept(RRab)	$\sigma$ (RRab)	slope(RRc)	intercept(RRc)	$\sigma$ (RRc)
LMC							
PC	max	$1.505 \pm 0.018$	$0.654 \pm 0.004$	0.116	$0.770 \pm 0.032$	$0.638 \pm 0.015$	0.084
	min	$0.093 \pm 0.019$	$0.716 \pm 0.005$	0.116	$0.604 \pm 0.041$	$0.836 \pm 0.020$	0.109
AC	max	$-0.361 \pm 0.003$	$0.592 \pm 0.002$	0.091	$-0.089 \pm 0.014$	$0.312 \pm 0.007$	0.091
	min	$0.049 \pm 0.003$	$0.651 \pm 0.003$	0.114	$0.411 \pm 0.017$	$0.357 \pm 0.008$	0.111
SMC							
PC	max	$1.768 \pm 0.053$	$0.705 \pm 0.012$	0.097	$0.536 \pm 0.228$	$0.529 \pm 0.100$	0.079
	min	$0.055 \pm 0.058$	$0.725 \pm 0.013$	0.099	$0.472 \pm 0.265$	$0.823 \pm 0.116$	0.091
AC	max	$-0.370 \pm 0.007$	$0.594 \pm 0.006$	0.074	$-0.312 \pm 0.078$	$0.450 \pm 0.039$	0.074
	min	$0.067 \pm 0.010$	$0.660 \pm 0.008$	0.098	$0.172 \pm 0.098$	$0.530 \pm 0.050$	0.094

observed for fundamental mode RR Lyrae stars as described in the next subsection. We also comment on the selection of 2.5 days as break period in the subsection describing the robustness of our analysis.

#### 4.2 Period-Color & Amplitude-Color relations for RR Lyrae variables

A study of MACHO RRab stars in the LMC was carried out by Kanbur & Fernando (2005). This suggested that for MACHO  $(V-R)$  colors, PC and AC relations are flat at minimum light (albeit with considerable scatter), but have a significant slope at maximum light. We extend this analysis to RR Lyrae stars in the LMC and SMC based on the OGLE-III survey. The plots of PC and AC relations for RRab and RRc stars in the Magellanic Clouds are presented in Fig. 5 and Fig. 6 respectively, while the results of fitting a linear regression to these PC and AC relations are summarized in Table 5. We find evidence to support a very shallow PC relation at minimum light in the  $(V-I)$  color for both LMC and SMC RRab stars. Similarly, the AC relation at minimum light is flat, while it has a significant negative slope at maximum light. Higher amplitude stars are driven to bluer colors at maximum light, like FO Cepheids in the Magellanic Clouds, and in line with equation (1).

We note that though the PC relation at minimum light is very shallow, there is considerable scatter. For the LMC/SMC the intercept of the PC relation at minimum light is  $0.716 \pm 0.005/0.725 \pm 0.013$ , with a standard deviation on this relation of  $0.116/0.099$ , respectively. Since the PC relation is not exactly flat hence a mean value at minimum light is  $0.692 \pm 0.034$  and  $0.711 \pm 0.043$  for LMC and SMC, respectively. We emphasize on this because there are a number of ways to define the minimum  $(V-I)$  color. Our definition is quite specific and given in Section 3 and is intended to capture the physics of the pulsation.

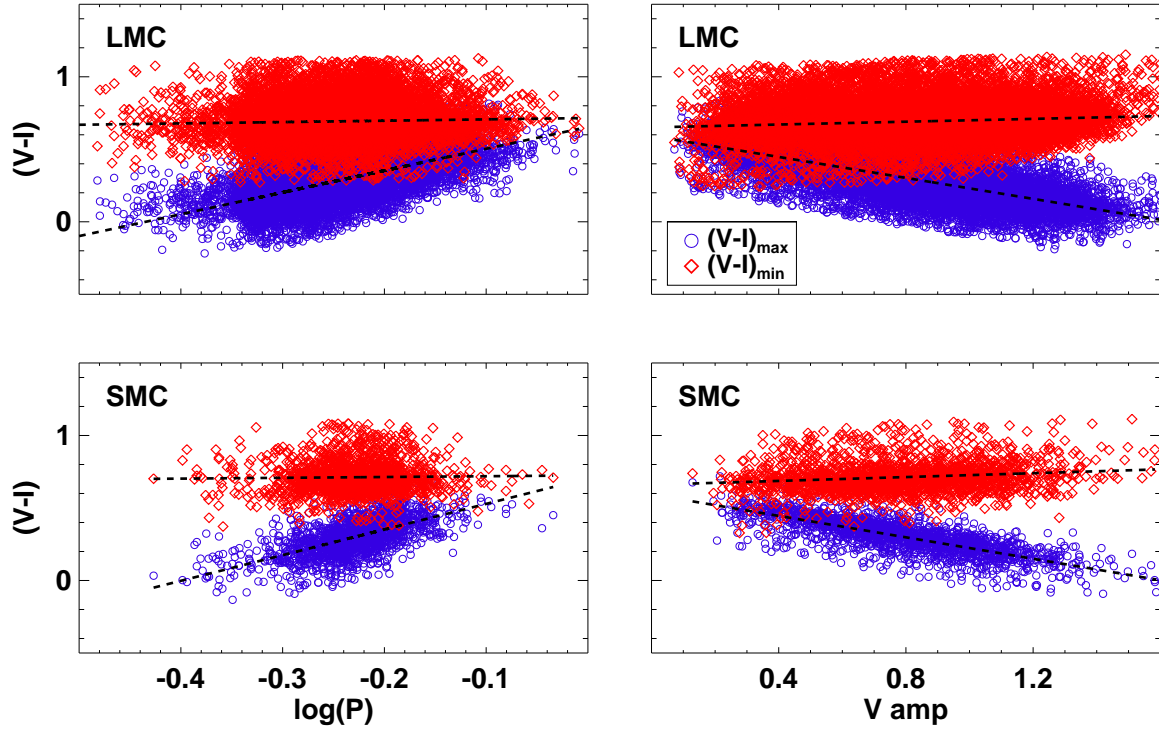
Previous work has suggested a common true intrinsic color for RRab stars at minimum light  $(V-I)_{min,0} = 0.57$  with an rms scatter of 0.025 mag (Day et al. 2002) and  $(V-I)_{min,0} = 0.58 \pm 0.02$  mag (Guldenschuh et al. 2005). These results use small samples of Galactic field RRab stars. Our work involves 17334/1917 stars in the LMC/SMC, and in terms of scatter in the PC relation produces very

similar results to the MACHO data (Kanbur & Fernando 2005). However, the definition of the minimum light color in Day et al. (2002) and Guldenschuh et al. (2005) is different to our analysis. They have computed the colors at minimum light via an arithmetic mean of the observed  $(V-I)$  data values, having phases between 0.5 and 0.8. Hence, instead of a mean, if we consider a true minimum of  $(V-I)$  data values, our results are identical to those obtained using the definition  $V_{min} - I_{phmin}$ . In case of all the definitions, there is little difference in the minimum light color between the LMC and SMC. Further, and perhaps more importantly, the scatter in the PC relation is quite large, indicating either intrinsic scatter due to unknown causes, or the presence of significant extinction that is not accounted for.

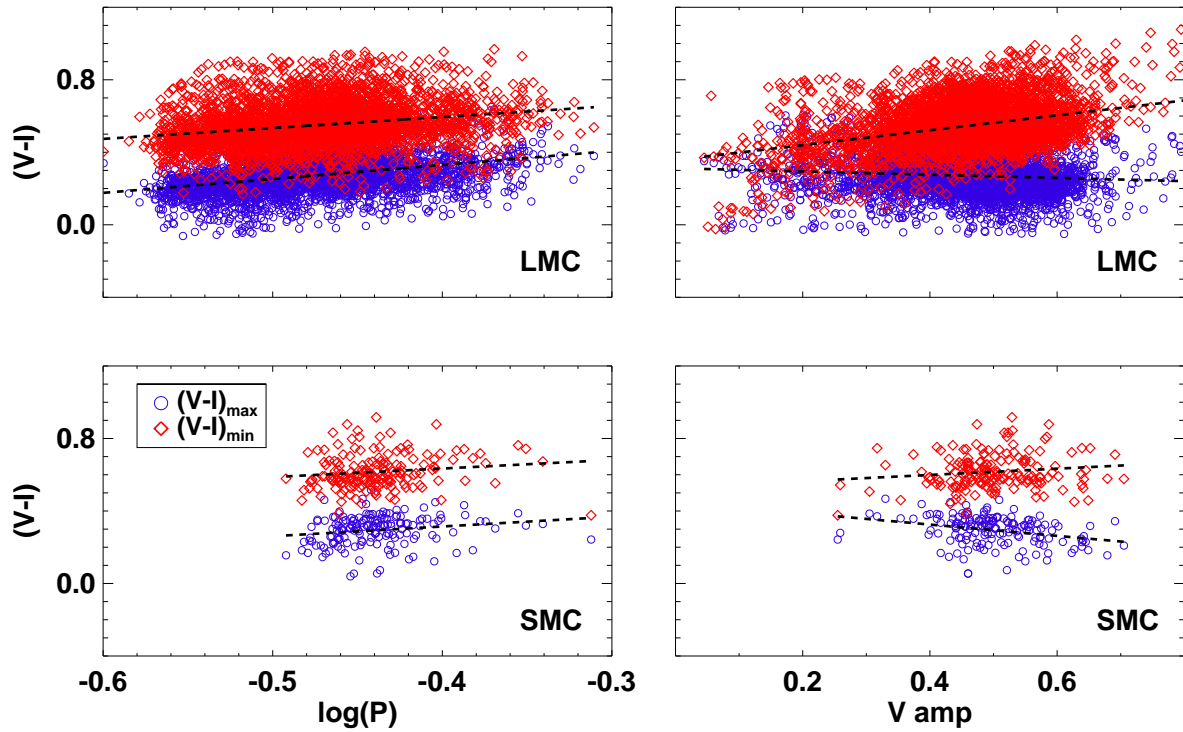
We note that FO Cepheids in the Magellanic Clouds with  $P < 2.5$  days behave in a similar way to RRab stars in terms of PC and AC relations at maximum and minimum light. Both groups of stars have a flat or relatively shallow PC relation at minimum light and a corresponding flat or shallow AC relation at maximum light. This flatness goes away for both PCAC relation at maximum light. One possibility to explain this is that FO Cepheids have hotter effective temperatures than FU Cepheids, hence the HIF in a FO Cepheid lies much further out in the mass distribution (Kanbur 1995) causing the HIF and photosphere to be engaged at minimum light in a similar way to RRab stars. If this is confirmed by further theoretical calculations, this will provide strong evidence that the HIF/stellar photosphere theory outlined in this and earlier papers is consistent with the observations.

The interesting thing that we observed for RRc stars is that the features we observe for RRab stars disappear for both PC and AC relations in the LMC and SMC. Recently Layden, Anderson & Husband (2013) have provided a PC relation for a small sample of field RRc stars as  $(V-I)_{min,0} = 0.225 + 0.536P$ , with an rms scatter of 0.073. Our analysis suggests  $(V-I)_{min} = 0.289 + 0.768P$  (with a standard deviation of 0.109) for LMC and  $(V-I)_{min} = 0.297 + 0.519P$  (with a standard deviation of 0.102) for SMC. Based on the PC and AC relations presented in Fig. 6 and Table 5, the RRc PC relation at minimum light is no longer flat but instead both PC and AC relations follow similar trajectories at maximum and minimum light. Since RRc stars





**Figure 5.** PC and AC relations for LMC and SMC RRab stars at maximum and minimum light. The dashed lines represent the best fit relations to the data.



**Figure 6.** PC and AC relations for LMC and SMC RRc stars at maximum and minimum light. The dashed lines represent the best fit relations to the data.

have in general hotter effective temperatures than RRab stars, RRc stars will have HIF's even further out in the mass distribution as compared to RRab stars. The stellar photosphere will therefore be located quite high up the HIF. Further theoretical work will be needed to investigate this possibility.

We also notice that the PC relations at maximum and minimum light for RRc stars in both the LMC and SMC are nearly parallel to each other. This is also true for FO Cepheids in the LMC.

### 4.3 Robustness

The assumptions behind the  $F$ -test are independence between observations, homoskedasticity and normality of residuals. It's clear that the observations are indeed independent of each other. We do see a trend in the residuals from our straight line regressions but that is because the data are consistent with two lines. There is some reduction in scatter of the residuals at periods close to 10 days but nothing to suggest that this would affect the distribution of the  $F$  statistic under the null hypothesis. Also qq plots of the residuals do exhibit normality except perhaps at the extrema but our results hold true if these outliers on the qq plots are removed (Kanbur & Ngeow 2004).

Our results do not change significantly even if we do not remove any outliers or if we do a  $2\sigma$  clipping. Further, moving the short period break from 2.5 days to 3.5 or 4.5 days makes the various nonlinearities slowly go away for SMC FU Cepheids. Similarly, for Magellanic Clouds FO Cepheids, the slope variations are no longer statistically significant if we shift the break point to 1 or 1.5 days. A similar situation holds for the longer period breaks at 10 days.

Further, our choice of applying a break period around 10 days is motivated, in part, by the way the structure of Cepheid light curves changes with period around a period of 10 days (Simon & Lee 1981). Sharp breaks are seen in the way the Fourier parameters change with period at 10 days.

In a similar way, we can look at the plots of Fourier parameters ( $R_{21}$ ,  $R_{31}$ ,  $\phi_{21}$  and  $\phi_{31}$ ) against  $\log(P)$  for SMC Cepheids, based on the Fourier decomposition method (Simon & Lee 1981). These plots for SMC FU & FO Cepheids light curves in the  $I$ -band, are displayed in Fig. 7 and Fig. 8, respectively. In the case of SMC FU Cepheids, the Fourier amplitude parameters presented in Fig. 7, show a maximum around 2.5 days ( $\log(P) = 0.4$ ). The Fourier phase parameters do not exhibit such patterns around 2.5 days but all parameters follow the change in progressions at 10 days. Fig. 8 presents the way Fourier parameters change with periods around 2.5 days for FO SMC Cepheids. We see that there is a local minimum around  $\log(P) = 0.4$ , especially for the plot of  $R_{21}$  against  $\log(P)$ . Similar changes are observed in the Fourier parameters for LMC FO Cepheids.

Resonances in the normal mode spectra of Cepheids have provided a convincing explanation for the Cepheid bump Progression ( $P_2/P_0 \approx 0.5, P_0 = 10$  days) and support for other features in Cepheid light curves (Simon & Lee 1981; Andreasen & Petersen 1987). Similarly, Soszynski et al. (2008) mentioned resonance features for the first overtone pulsators at periods of about 0.35 and 3 days in OGLE-III data. The second feature has been interpreted as the signature of a 2:1 resonance between the first and

fourth overtones (Antonello & Poretti 1986). Our ideas offer a detailed scenario through which Period-Color relations and hence Period-Luminosity relations may change suddenly. These ideas are not mutually exclusive of the resonance scenario. However, we note that our work can also be extended to RR Lyraes. There are some indications of resonances in the RR Lyrae mode spectra, but nothing as yet, that seems to stand out as significantly as the Cepheid  $P_2/P_0$  resonance. The similarity in terms of PC relations at minimum light between FO Cepheids and FU RR Lyraes provides strong support for our ideas but again does not impinge on the effect of resonances in the normal mode spectra.

## 5 DISCUSSION AND CONCLUSIONS

We have provided empirical evidence for the existence of a number of nonlinearities in PCAC relations at maximum and minimum light for Cepheids and RR Lyraes in the Galaxy and Magellanic Clouds. Specifically:

- (i) The Galactic FU Cepheid PC relations at maximum and minimum light, together with the AC relation at minimum light, exhibit a break at a period of 7 days.
- (ii) The Galactic FU Cepheid PC relation at maximum light is flat for periods greater than 7 days, and is significantly different to the PC relation at maximum light for shorter period counterparts.
- (iii) At minimum light, neither the short nor long period Galactic FU Cepheids display a flat PC relation.
- (iv) The Galactic AC relation at minimum light is such that higher amplitude stars are driven to cooler temperatures and hence redder colors at minimum light.
- (v) For FU Cepheids in the Magellanic Clouds, there are PCAC breaks at 10 days at both maximum and minimum light. In both cases the PC relation at maximum light for longer period Cepheids is much shallower than for shorter period Cepheids. Similarly, in both Clouds the AC relation at minimum light goes from having a slope consistent with zero for shorter period ( $P < 10$  days) Cepheids to having a positive slope for longer period Cepheids. In addition the slope of the AC relation at maximum light gets significantly lower across the 10 day transition.
- (vi) In case of SMC FU Cepheids, we observe significant breaks in PC and AC relations around 2.5 days at maximum and minimum light.
- (vii) In the case of FO Cepheids in Magellanic Clouds, there are PCAC breaks at both maximum and minimum light at a period around 2.5 days. More specifically, for the LMC FO Cepheids, the PC relations at maximum and minimum light are very shallow for shorter period ( $P < 2.5$  days) Cepheids but much steeper for longer ( $P > 2.5$  days) period stars. This is also true for the SMC PC relation at minimum light. However the SMC PC relation at maximum light is quite steep for shorter period FO Cepheids ( $P < 2.5$  days) and becomes even steeper for longer period FO Cepheids. For both LMC and SMC, the slope of the short period ( $P < 2.5$  days) AC relation at minimum light is very shallow and becomes much steeper for long period FO Cepheids. At maximum light the slope becomes less steep as one goes across the 2.5 days break period.
- (viii) RRab stars in Magellanic Clouds have a flat PC relation at minimum light with a corresponding relation

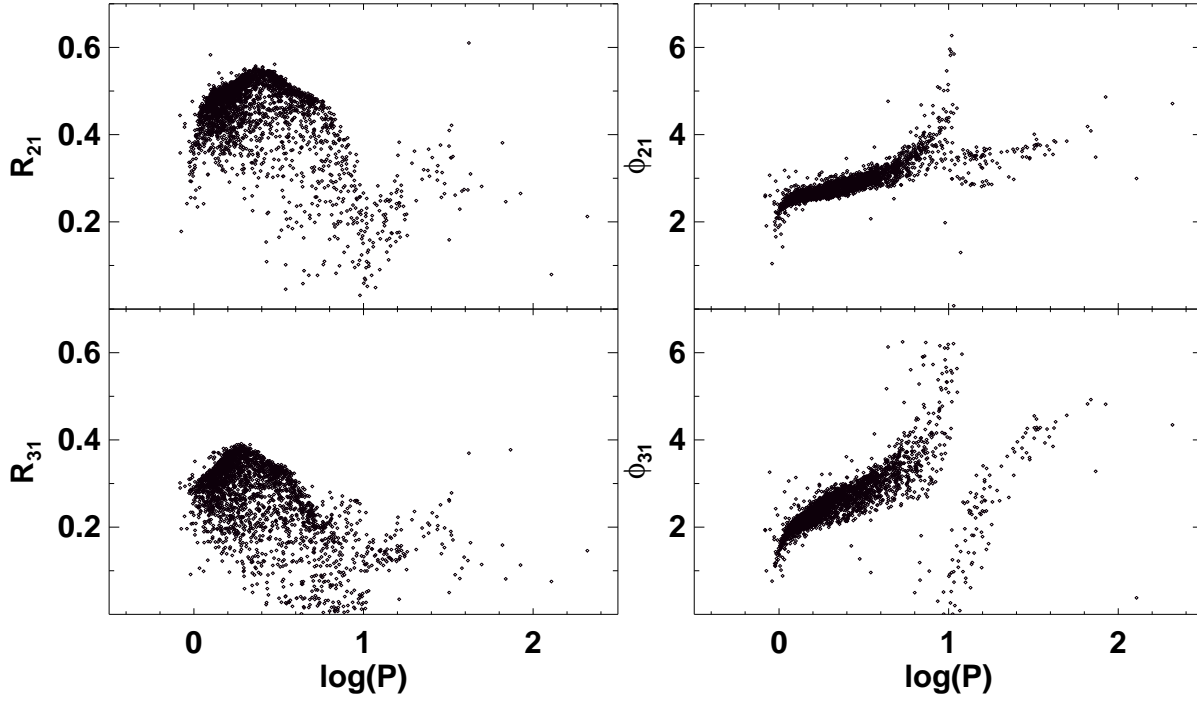


Figure 7. Fourier parameters derived from Fourier decomposition technique for fundamental mode Cepheids in SMC using the *I*-band data.

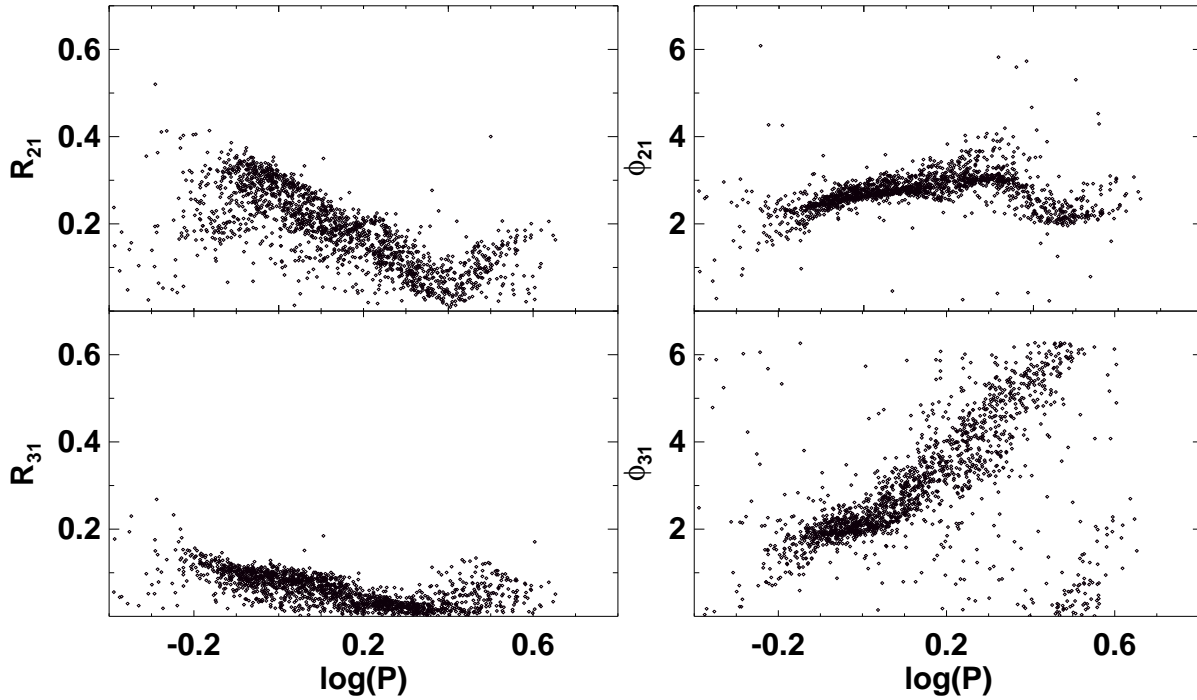


Figure 8. Fourier parameters derived from Fourier decomposition technique for first overtone Cepheids in SMC using the *I*-band data.

between amplitude and color at maximum light such that higher amplitude stars have a hotter temperature and a bluer color at maximum light. Moreover, the flat PC relations at minimum light suggest that the the  $(V - I)_{min}$  color in the LMC and SMC are consistent with each other to within the quoted errors. Nevertheless, the dispersion of the PC relations around this minimum light color is quite significant.

(ix) The RRc stars in both the Magellanic Clouds do not exhibit such a flat PC relation at minimum light.

(x) We note that the FO Cepheids in the Magellanic Clouds have a flat PC relation at minimum light just like RRab stars in the Magellanic Clouds.

In the following subsection we provide a possible theoretical scenario for some of these results. The core of our theory relies on the interaction of the HIF and stellar photosphere and the properties of the Saha ionization equation that is thought to determine ionization equilibrium in such stars.

### 5.1 A possible theoretical interpretation

The stellar photosphere and HIF are not generally co-moving during a pulsation cycle. The two can be engaged when the stellar photosphere occurs at the base of the HIF and this engagement is sudden, i.e. they are either engaged or not engaged. When they are engaged the temperature of the stellar photosphere is the temperature at which hydrogen ionizes. Or rather the temperature of the stellar photosphere is the temperature at which a large enough fraction of hydrogen is ionized to cause a substantial increase in opacity sufficient to prevent the photosphere from moving further in the mass distribution of the star. Consequently, the color of the star is the color corresponding to the temperature of the stellar photosphere for that particular star. When this engagement occurs at low densities the PC relation is flat or shallow. When this engagement occurs at high densities, a higher temperature is needed in order to achieve the same level of hydrogen ionization and thus a large enough opacity to prevent the stellar photosphere from moving in any further in the mass distribution. When the stellar photosphere and HIF are not engaged, the temperature of the stellar photosphere is again dependent on period and global stellar parameters.

Changes in PC relations at maximum and minimum light can explain changes in AC relations due to equation (1). Thus (ii) occurs because for FU Cepheids, the HIF and stellar photosphere are engaged at low densities only at maximum light leading to a flat PC relation at maximum light from a period of about 7 days onwards. Similarly, (iv) occurs because of equation (1) and the fact that the PC relation at maximum light is flat.

FU Cepheids in the Magellanic Clouds have a different ML relation that changes the relative location of the HIF and stellar photosphere such that the two are engaged at maximum light, at low densities only for periods greater than 10 days. Thus for Magellanic Clouds FU Cepheids with periods longer than 10 days, higher amplitude stars are driven to cooler temperatures and hence redder colors at minimum light – this explains (v).

Since RR Lyraes have hotter effective temperatures

than Cepheids, the HIF lies further out in the mass distribution so that it is engaged with the photosphere throughout the pulsation cycle. However, the engagement only occurs at low densities and temperatures at minimum light (Kanbur 1995).

As the star brightens from minimum light, the HIF is pushed further out in the mass distribution. Thus the temperature of the stellar photosphere has to be hotter in order to get a greater fraction of hydrogen ionization in order to get a high enough opacity to get to a given optical depth. Thus the color at maximum light is not flat: higher temperatures are needed in order to get the required hydrogen ionization fraction to achieve a high enough opacity to block further progress of the stellar photosphere into the mass distribution. These required higher temperatures are dependent on period and global stellar parameters. Thus the PC relation has a positive slope with a definite relation between amplitude and color at maximum light such that higher amplitude stars are driven to hotter temperatures and bluer colors at maximum light because of equation (1). This explains (vii) and (viii).

This behavior (a flat PC relation at minimum light) is not seen for RRc stars. Following the reasoning in Kanbur (1995) we postulate that because overtone stars are generally hotter than fundamental mode stars, their HIF lies even further out in the mass distribution and the stellar photosphere can get some way "up" the HIF before the high opacity prevents any further incursion. The temperature of the stellar photosphere would then be more dependent on global stellar parameters such as the period. This suggests a possible explanation for (ix).

We see from Table 4 that FO Cepheids in the Magellanic Clouds with periods shorter than 2.5 days behave in a similar way to RRab stars. They have a flat PC relation at minimum light. Correspondingly they have a well defined AC relation at maximum light and an AC relation with a slope consistent with zero at minimum light. This is again in accord with equation (1). FO Cepheids with periods shorter than 2.5 days are hotter than FU Cepheids and their HIF lies further out in the mass distribution than FU Cepheids. Hence in terms of the qualitative features of PCAC relations at maximum and minimum light, they behave just like RRab stars. FO Cepheids with periods greater than 2.5 days have cool enough temperatures and higher  $L/M$  ratios so that the stellar photosphere is disengaged from the HIF and the temperature and hence color associated with the stellar photosphere would be more dependent on global stellar parameters, thus explaining (x). The theoretical concepts presented in this paper build on the theoretical calculations carried out in Simon, Kanbur & Mihalas (1993), Kanbur (1995), Ngeow & Kanbur (2006), Kanbur, Ngeow & Feiden (2007) and Kanbur et al. (2010). In future work we plan a detailed theoretical study to determine if these ideas are consistent with the observations.

### 5.2 Implications for the Cepheid Period-Luminosity relation

The Cepheid Period-Luminosity (PL) relation is important for CMB independent estimates of Hubble's constant. Since the PL relation is just the projection of the Cepheid Period-Luminosity-Color (PLC) relation on the period and lumi-

osity (or magnitude) planes, hence PC relations can affect PL relations through this PLC relation. Both of the PL and PC relations at mean light are obtained by the numerical average of the corresponding PL and PC relations at every phase during a pulsation period, therefore, changes in PC relations at particular phases can indeed have effects on the mean light PC and PL relation. One way to understand PC relations at mean light in greater depth is to study them as a function of phase, such as at maximum and minimum light. Thus this approach can also lead to insights into possible nonlinearities in PC relations at mean light (e.g., see Kanbur & Ngeow 2004). This is demonstrated here by the finding that the SMC FU Cepheid PCAC relations exhibit a highly significant break at a period of 2.5 days. This is consistent with the findings of Bauer et al. (1999) who found a break in the mean light SMC PL relation for the FU Cepheids using EROS data.

We also note that our results pertain to the *V*- and *I*-bands. At longer wavelengths such effects become harder to measure because amplitudes are smaller - temperature fluctuations lead to smaller effects in color as the wavelength increases due to the black body law. However any effects that happen at optical depths close to the photosphere could indeed be affected by some of the physics discussed here - for example, colors at longer wavelengths that can be influenced by opacity effects related to molecules.

### 5.3 Further work

We note that even though we have strong evidence for breaks in PCAC relations at a number of phases using the *F*-test and found these to be consistent with a plausible theoretical scenario, more work needs to be done in order to be definitive about these results both from a theoretical and observational point of view. More specifically, it is important to carry out a further range of statistical tests, such as the testimator (Kanbur et al. 2007) and random walk (Koen, Kanbur & Ngeow 2007), to be absolutely sure of our observational results and stellar pulsation, structure and evolution modeling to further understand the nature of the HIF-stellar photosphere interaction. We note that further investigation is needed before the SMC FU Cepheid and MC FO nonlinearities can be accommodated within the framework of our stellar photosphere/HIF interaction theory. Thus one way to understand possible nonlinearities in Cepheid PL relations is through the use of multiphase PC and PL relations as suggested here.

### ACKNOWLEDGMENTS

AB is thankful to the Council of Scientific and Industrial Research (CSIR), New Delhi for a Junior Research Scholarship (JRF). This work is supported by the grant provided by Indo-U.S. Science and Technology Forum under the Joint Center for Analysis of Variable Star Data. SMK thanks SUNY Oswego for partial funding of research visits to the University of Delhi where some of this work was completed. CCN thanks the funding from Ministry of Science and Technology (Taiwan) under the contract NSC101-2112-M-008-017-MY3. The authors would like to thank Marcella

Marconi, Ata Sarajedini, Sukanta Deb and Rachel Wagner-Kaiser for helpful discussions.

### REFERENCES

- Andreasen G. K., Petersen J. O., 1987, *A&A*, 180, 129
- Antonello E., Poretti E., 1986, *A&A*, 169, 149
- Bart M. L., 1982, *IMA J. Num. Analysis*, 2, 241
- Bauer F. et al., 1999, *A&A*, 348, 175
- Berdnikov L. N., 1987, *Peremennye Zvezdy*, 22, 530
- Berdnikov L. N., 1992, *Soviet Astronomy Letters*, 18, 130
- Berdnikov L. N., 2008, *VizieR Online Data Catalog*, 2285, 0
- Berdnikov L. N., Ignatova V. V., Pastukhova E. N., 1998, *Astronomical and Astrophysical Transactions*, 15, 81
- Berdnikov L. N., Turner D. G., 2001, *Astronomical and Astrophysical Transactions*, 19, 689
- Berdnikov L. N., Turner D. G., 2004a, *Astronomical and Astrophysical Transactions*, 23, 395
- Berdnikov L. N., Turner D. G., 2004b, *Astronomical and Astrophysical Transactions*, 23, 599
- Berdnikov L. N., Vozyakova O. V., 1995, *Astronomy Letters*, 21, 308
- Berdnikov L. N., Yakubov S. D., 1993, *Peremennye Zvezdy*, 23, 47
- Day A. S. et al., 2002, *PASP*, 114, 645
- Deb S., Singh H. P., 2009, *A&A*, 507, 1729
- Deb S., Singh H. P., 2010, *MNRAS*, 402, 691
- Deb S., Singh H. P., 2014, *MNRAS*, 438, 2440
- Fernie J. D. et al., 1995, *Information Bulletin on Variable Stars*, 4148, 1
- Guldenschuh K. A. et al., 2005, *PASP*, 117, 721
- Haschke R., Grebel E. K., Duffau S., 2011, *AJ*, 141, 158
- Kanbur S. M., 1995, *A&A*, 297, L91
- Kanbur S. M., Fernando I., 2005, *MNRAS*, 359, L15
- Kanbur S. M. et al., 2010, *MNRAS*, 408, 695
- Kanbur S. M. et al., 2007, *PASP*, 119, 512
- Kanbur S. M., Ngeow C.-C., 2004, *MNRAS*, 350, 962
- Kanbur S. M., Ngeow C.-C., 2006, *MNRAS*, 369, 705
- Kanbur S. M., Ngeow C.-C., Buchler J. R., 2004, *MNRAS*, 354, 212
- Kanbur S. M., Ngeow C.-C., Feiden G., 2007, *MNRAS*, 380, 819
- Koen C., Kanbur S., Ngeow C., 2007, *MNRAS*, 380, 1440
- Layden A., Anderson T., Husband P., 2013, *ArXiv e-prints*, 1310.0549
- Ngeow C.-C., Kanbur S. M., 2006, *MNRAS*, 369, 723
- Petersen J. O., 1986, *A&A*, 170, 59
- Simon N. R., Kanbur S. M., Mihalas D., 1993, *ApJ*, 414, 310
- Simon N. R., Lee A. S., 1981, *ApJ*, 248, 291
- Soszyński I. et al., 2010a, *Acta Astronomica*, 60, 17
- Soszyński I. et al., 2010b, *Acta Astronomica*, 60, 165
- Soszyński I. et al., 2008, *Acta Astronomica*, 58, 163
- Soszyński I. et al., 2009, *Acta Astronomica*, 59, 1
- Tammann G. A., Sandage A., Reindl B., 2003, *A&A*, 404, 423
- Ulaczyk K. et al., 2013, *Acta Astronomica*, 63, 159